



Plane Wave – normal incidence

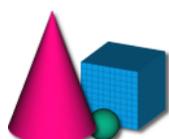
Introduction

The aim of the training given below is to point out main properties and behaviour of the plane wave propagating in an isotropic medium and incident normally at the media boundaries.

All examples used here were prepared in free CAD *QW-Modeller* for *QuickWave* and the models preparation procedure is described in separate documents. All examples considered herein are included in the *QW-Modeller* and *QuickWave* STUDENT Release installation as both, *QW-Modeller* and *QW-Editor* projects.

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Plane wave in a free space

Theory

A plane wave is the simplest form of the Maxwell's equations solution. Its name refers to the shape of a wave front (constant phase surfaces, perpendicular to the direction of propagation) of a propagating wave:

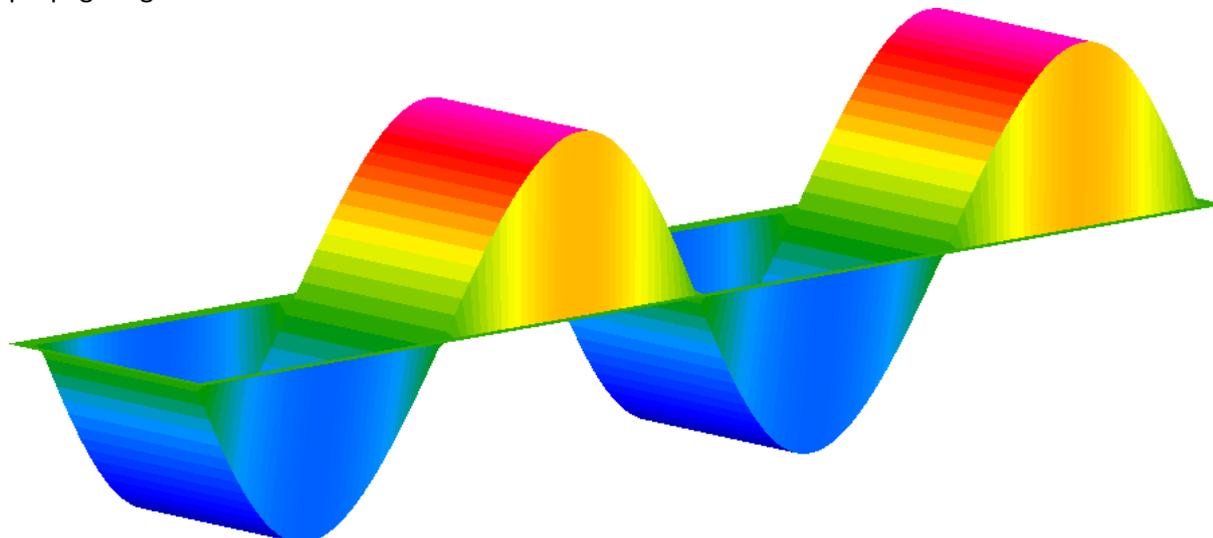


Fig 1 Plane wave in a free space.

The basic parameters of a plane wave are: wavelength λ , frequency f , and magnitude A . Wavelength is related to frequency and propagation speed (thus also the parameters of the medium in which wave propagates) according to the following formula:

$$f = \frac{v}{\lambda} = \frac{1}{\sqrt{\mu\varepsilon}} \frac{1}{\lambda}$$

where:

- ε - medium permittivity
- μ - medium permeability

There are four *main properties* of a plane wave:

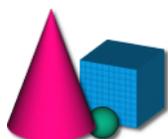
1. Propagation speed - depends on medium parameters:

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

where $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \cong 3 \cdot 10^8 \frac{m}{s}$ is a speed of light in vacuum, μ_r and ε_r stand respectively for the relative permeability and permittivity of a medium in which the wave propagates.

2. Electric and magnetic fields are perpendicular to the direction of propagation:

$$\vec{k} \perp \vec{E} \text{ and } \vec{k} \perp \vec{H}$$



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3. Electric and magnetic fields are perpendicular to each other:

$$\vec{E} \perp \vec{H}$$

4. Wave impedance is equal to intrinsic impedance of the medium:

$$Z_w = \frac{E_{\perp}}{H_{\perp}} = \sqrt{\frac{\mu}{\epsilon}} = Z_i$$

The above given formulae apply to a plane wave propagating in a lossless medium. When considering a plane wave in a lossy medium a general formula for sinusoidal plane wave is given with a complex notation as follows:

$$\vec{E} = \underline{\vec{E}}_0 e^{j\omega t - \vec{\gamma} \cdot \vec{r}}$$

where $\underline{\vec{E}}_0$ stands for a complex amplitude, ω is a pulsation, t – time, $\vec{r} = [x, y, z]$ – radius vector and γ – propagation constant, given with

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

where β is a phase constant and α an attenuation coefficient.

Paragraph objective

- 1) Study of the basic properties of a plane wave.
- 2) Observations of a wavelength change as a frequency function.
- 3) Observations of a wavelength change as a function of material parameters: permittivity and permeability.
- 4) Observations of field envelope changes as a function of medium losses.

Tutorials

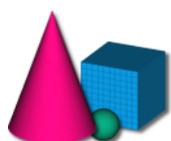
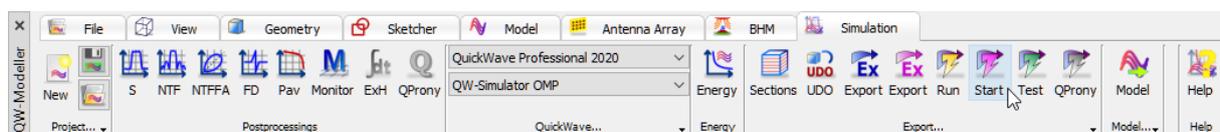
For the tutorials the basic *ppw1.QWpro* model and its modifications will be used. The description of how to create such model step by step can be found in a separate document.

Basic properties of a plane wave

The *ppw1.QWpro* model should be loaded to QW-Modeller and the electromagnetic simulation in *QuickWave* needs to be run. The simulation is run with default parameters of the model, by pressing



Start button from *Simulation* tab.



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QW-Simulator runs with Log window opened and Log Output tab active. Detailed description of all

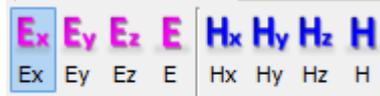


output comments written in the log window can be found in QuickWave manual. Press Fields button in 2D/3D Fields tab of QW-Simulator to open fields visualisation window and observe the propagating wave. Maximise the window for your convenience. In the window visible are: the structure and a cross section through the middle of the parallel plate line, in the plane parallel to the direction of wave propagation to enable convenient visualisation. Enable continuous display refresh by pressing



Dynamic icon (or by pressing "D" from keyboard). In the window title bar "/DYN" should now appear.

The field component that is currently displayed is E_x , which in the considered case stands for the electric field component parallel to the direction of wave propagation. According to Property 2 of a plane wave, electric and magnetic fields are perpendicular to the direction of wave propagation, that is why the E_x component is zero. Switch to E_z and then to H_y components by clicking the appropriate



buttons in the Components tab: E_x E_y E_z E H_x H_y H_z H . Those two components obey the Property 2 of a plane wave, thus when displayed the moving sinusoidal wave can be seen. We



recommend adjusting the magnitude of the displayed component using Adjust button. The exemplary display of the electric field component is shown Fig 2.

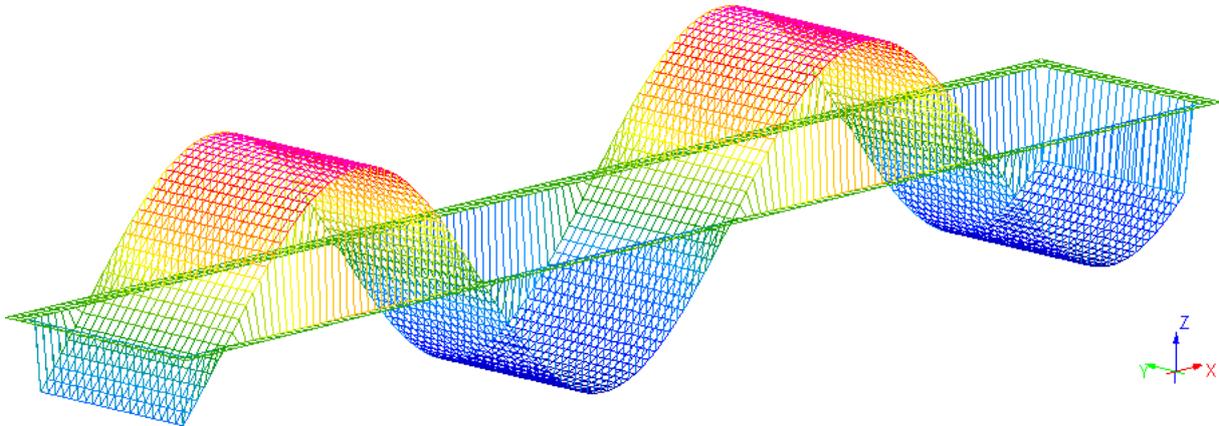
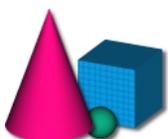


Fig 2 Example of E_z component of electric field.

The display confirms that the wave propagates in \vec{i}_x direction and the electric and magnetic fields are along \vec{i}_z, \vec{i}_y directions respectively, what proves the Property 3 of a plane wave.

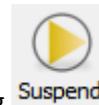
It should be reminded that the frequency set for this model is 4GHz, thus the wavelength in a free space is calculated as follows:

$$\lambda = \frac{v}{f} = \frac{c}{f} = 0.075 \text{ m} = 75 \text{ mm}$$





This value can be confirmed in the simulation. Press **Fields** button in *1D Fields* tab and then change



the display to show *Ez* component. Suspend the simulation by pressing **Suspend** button. Fig. 3 shows the display in a random simulation moment after suspending the calculation. To calculate the wavelength it is required to measure the distance between two nearest points of the same phase. It is done using markers (blue and red) and the distance is read on the left side of the window. In this case **X_shift=75.00 [mm]** what is consistent with the theoretical calculations.

The *QW-Simulator* can be now closed.

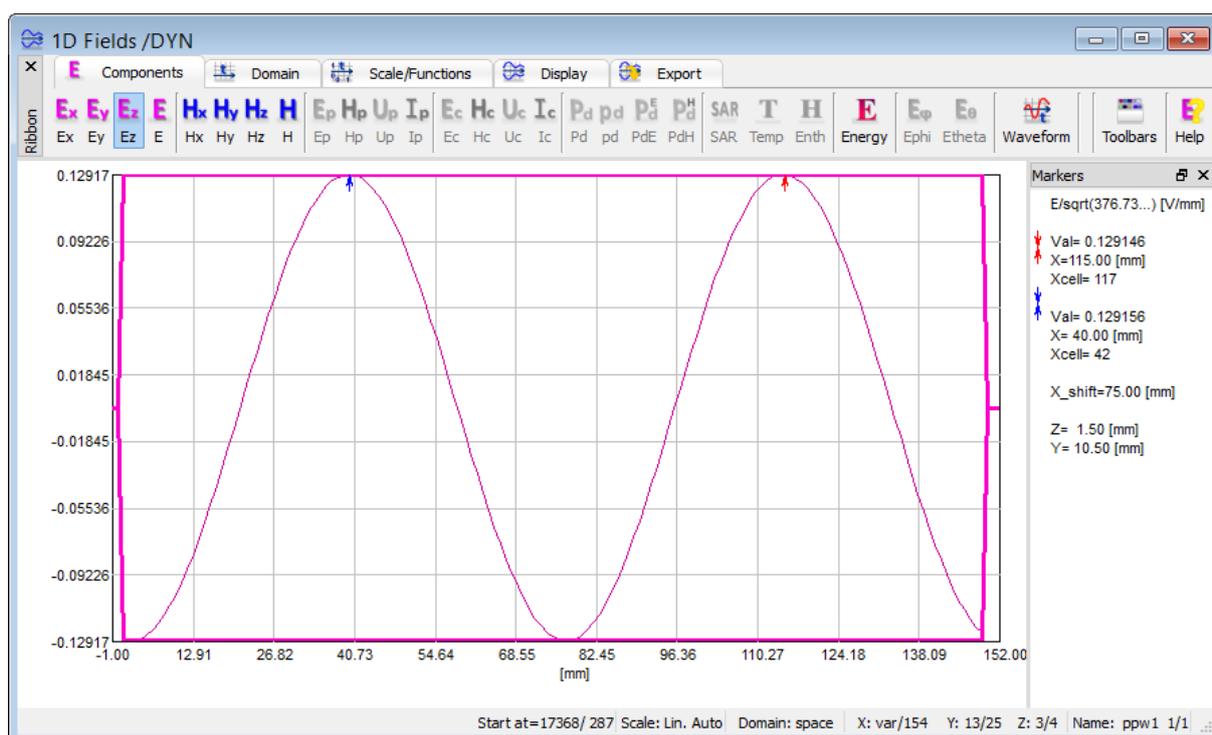


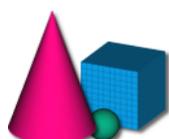
Fig 3 Example of 1D view of *Ez* component.

Relation between a frequency and a wavelength

The formula for a wavelength calculation has been confirmed with simulation. Now the dependence between a frequency and a wavelength will be investigated. For that purpose you can open a ready to run *ppw2.QWpro* file or open and modify the *ppw1.QWpro* example in *QW-Modeller*. If you are modifying *ppw1.QWpro* file, change the excitation parameters using dialogue windows of respective ports, which can be edited from the project tree (see Fig 4). Click over *Ports* (to unfold the menu) and double click over the *SourcePort* port to open its configuration dialogue (see Fig 5). In the *Parameters* tab we set the excitation frequency to 8 GHz. The same change should be done for *LoadPort* port.



Save the project with a new name and run simulation by pressing **Start** in *Simulation* tab.



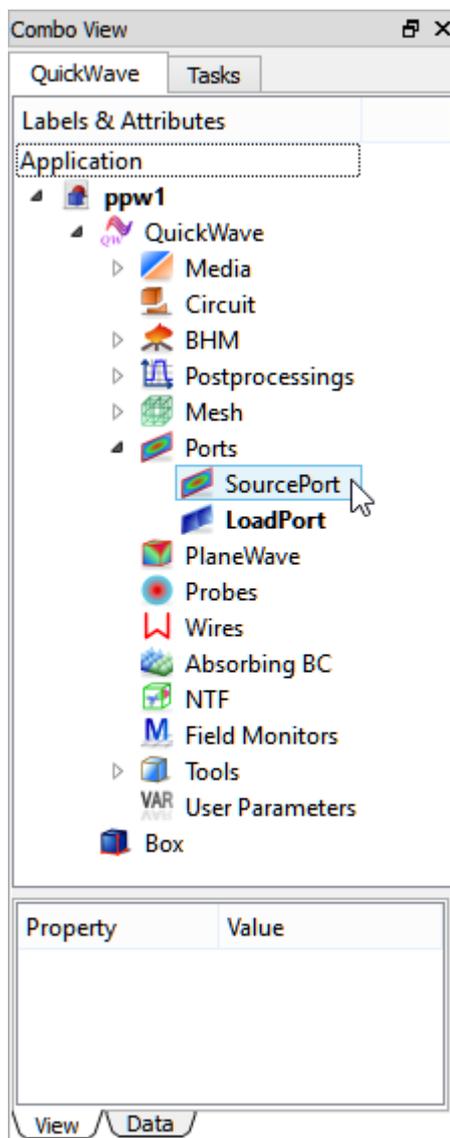
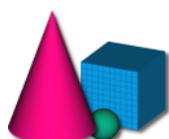


Fig 4 ppw1.QWpro project tree.



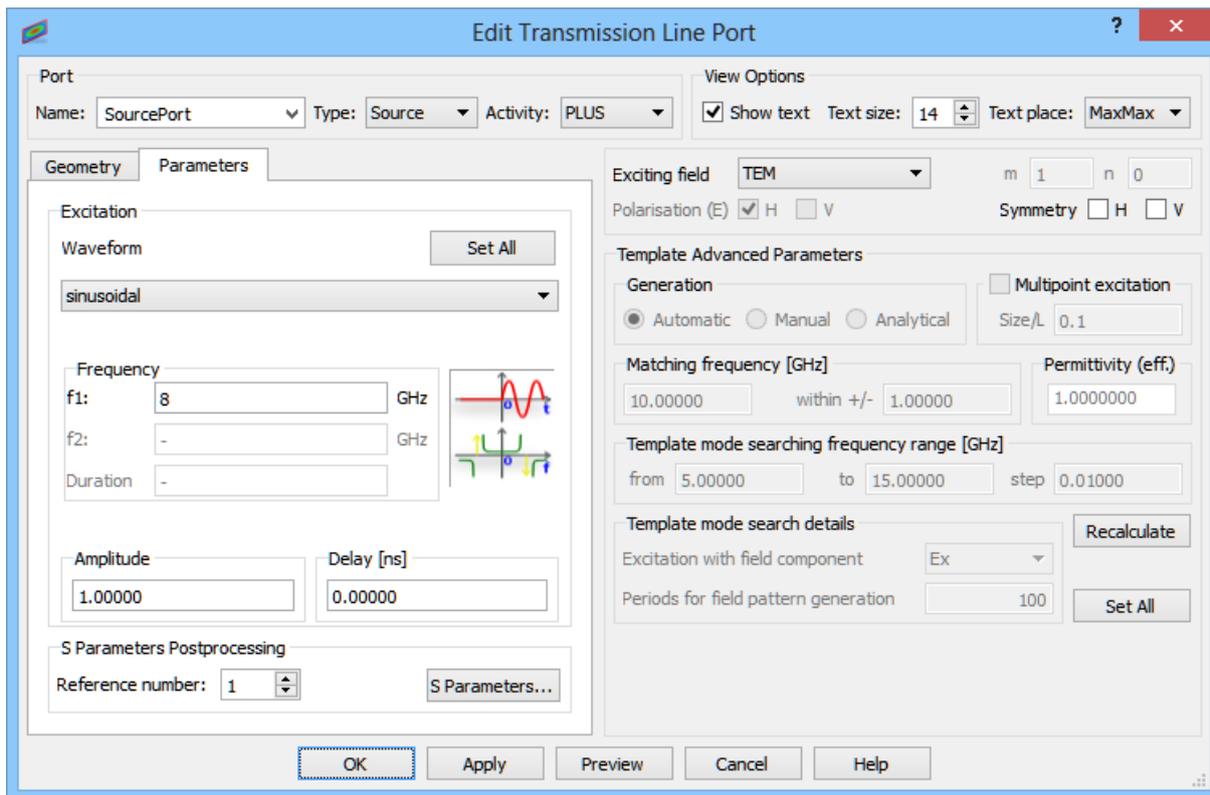
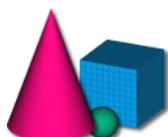


Fig 5 Port properties dialogue.

The wavelength should be measured as previously (open 1-D display window with  Fields button, pick the correct component, suspend the simulation, mark two points of the same phase). The result of this measurement is shown in Fig 6.



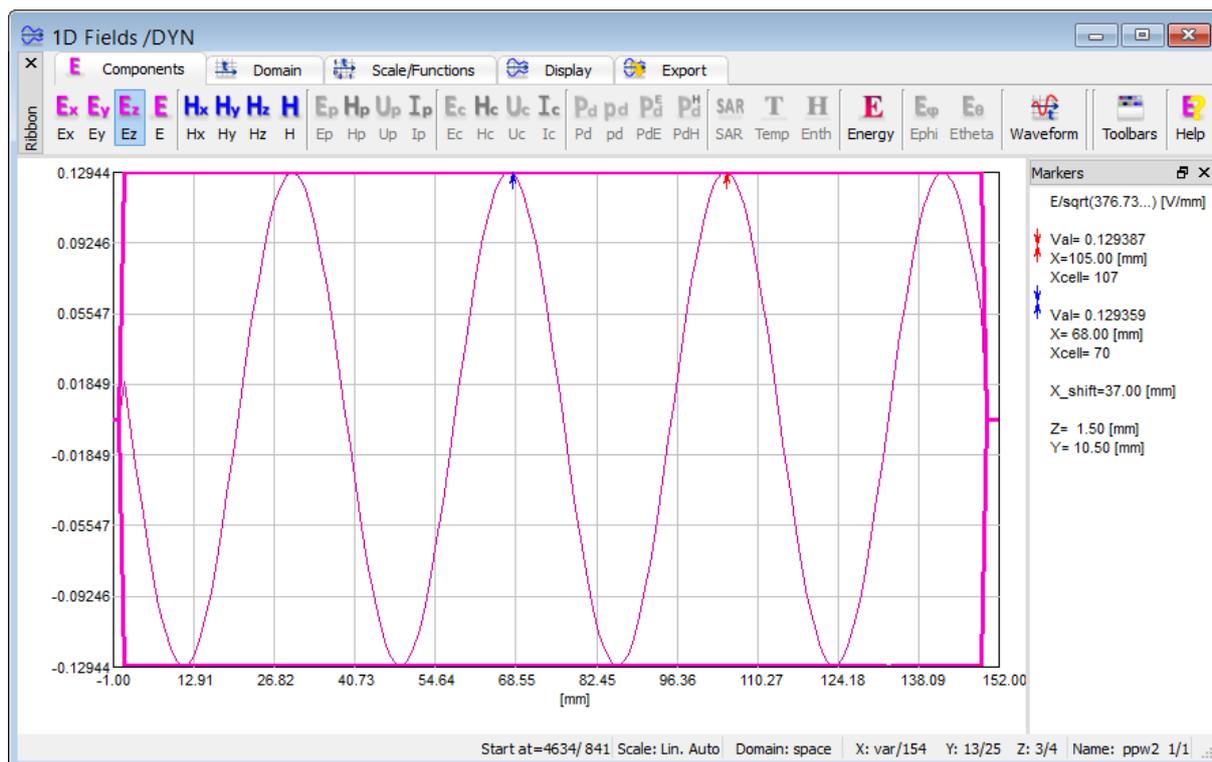


Fig 6 Measurement of wavelength at 8GHz.

The theoretical wavelength is:

$$\lambda = \frac{v}{f} = \frac{c}{f} = 0.0375 \text{ m} = 37.5 \text{ mm}$$

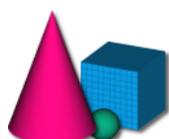
and it differs from the value calculated in simulation. This discrepancy is due to the space discretisation setting, which directly determines the accuracy of wavelength calculation. In the considered case the cell dimension in the direction of wave propagation equals 1 mm and that is the limit of wavelength calculation accuracy.

Relation between a wavelength and medium parameters (permittivity and permeability)

The relation between a wavelength and medium parameters can be observed using *ppw3.QWpro* file that needs to be loaded in QW-Modeller.

It is also possible to modify *ppw1.QWpro*, which for that purpose should be opened in QW-Modeller. To observe the relation between a wavelength and material properties with good accuracy it is

required to change the space discretisation. In the *Model* tab press  **Settings** icon and in the appearing dialogue set the cell size in the direction of wave propagation (X dir) to 0.25 mm (see Fig. 7).



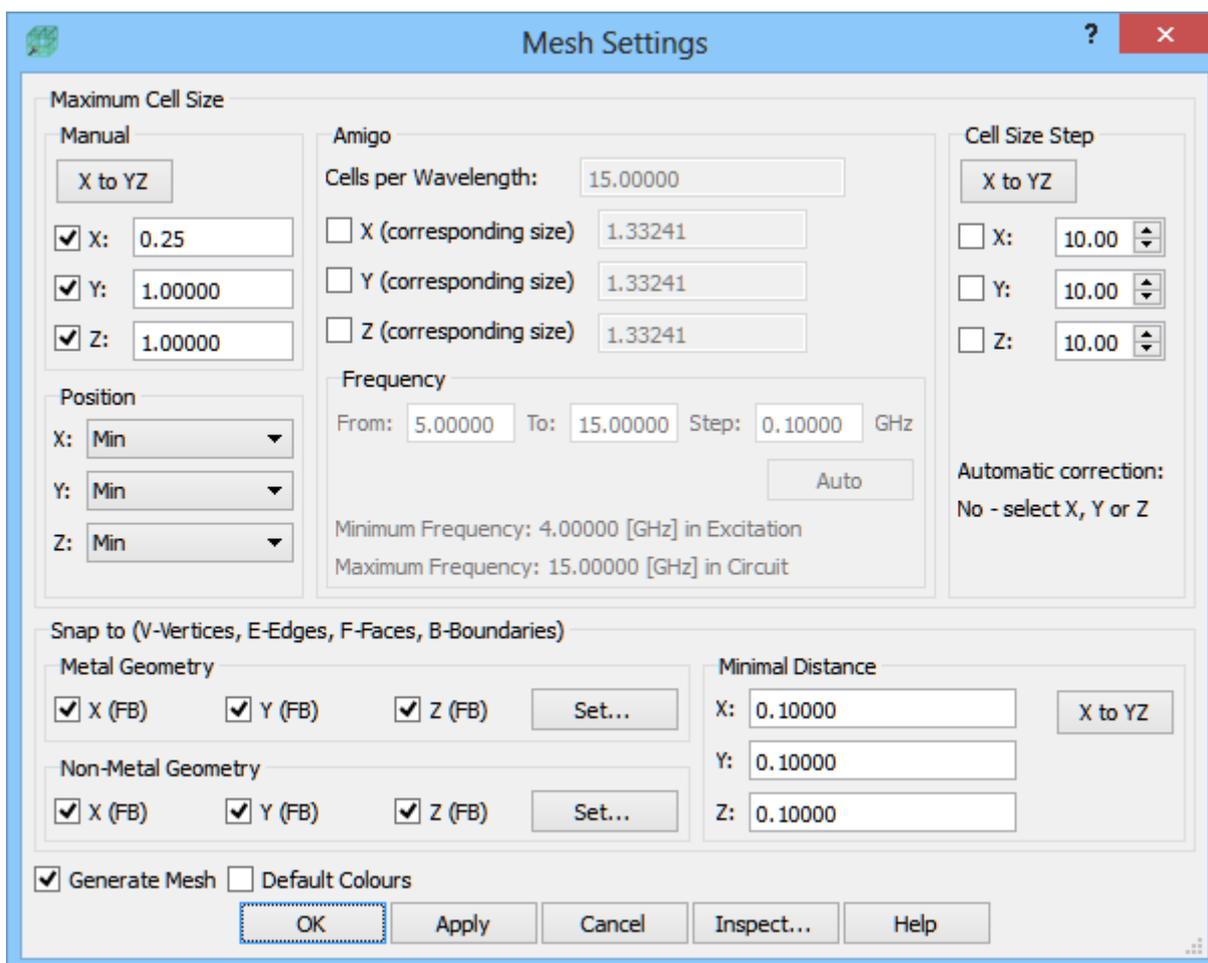
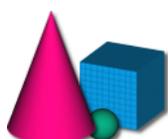


Fig 7 Dialogue window for mesh settings.

Now, the new medium in which the wave will propagate should be introduced and assigned. The procedure of doing this is as follows:



- In *Geometry* tab press **Media** button. The dialogue as in Fig 8 will be opened.
- Press **Add** button. A window for defining medium parameters, as in Fig.9 appears. Define the material name as “dielectric” and assign permittivity value of $\epsilon_r=4$. If required, material colour can be changed, using *Pen* option.



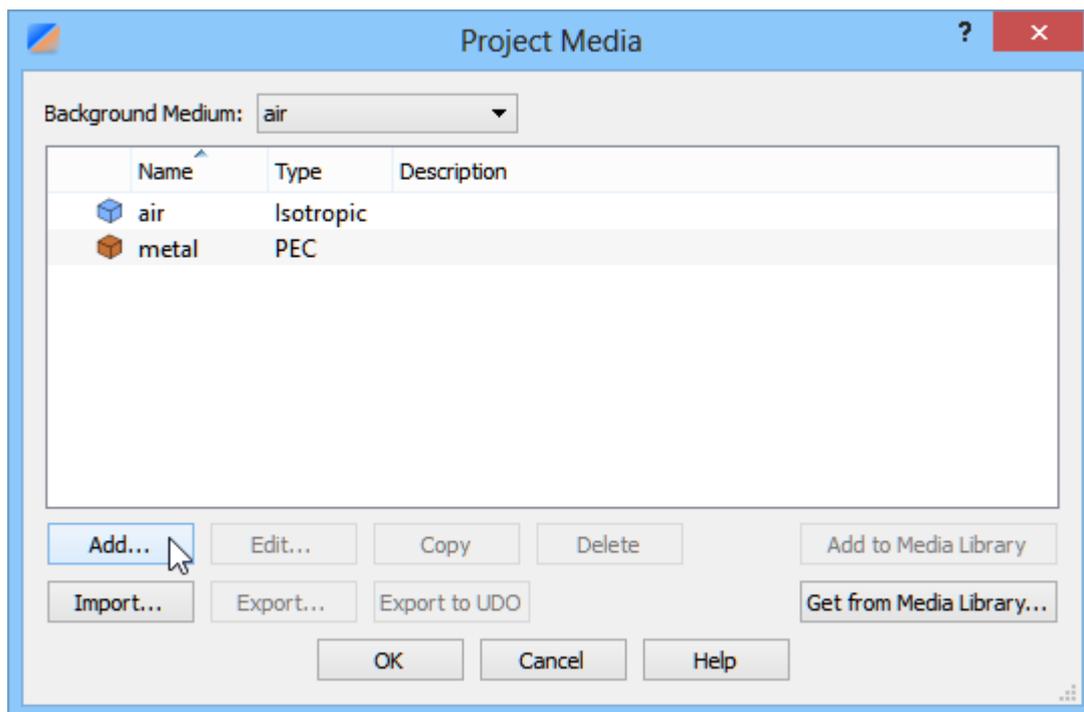


Fig 8 Dialogue window for materials editing.

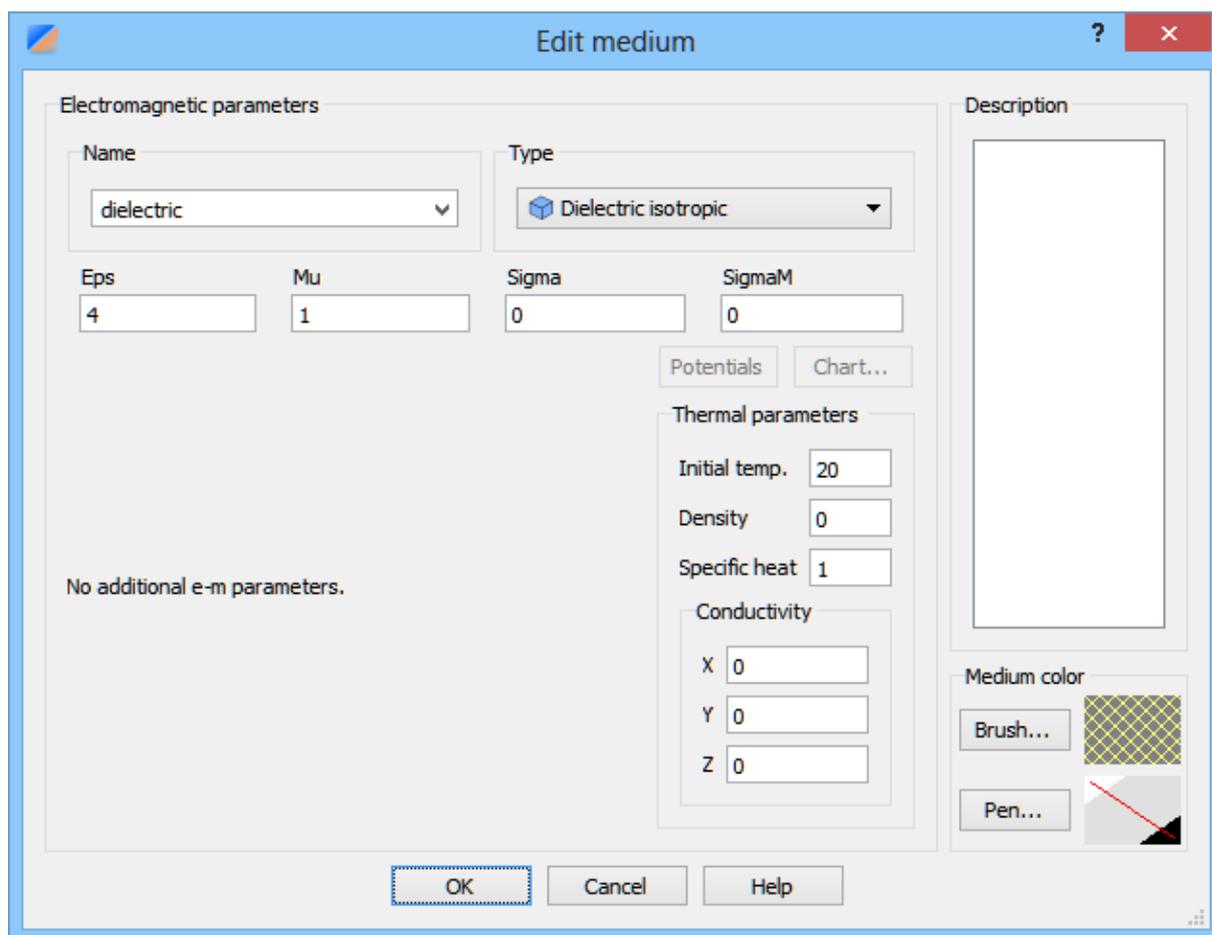
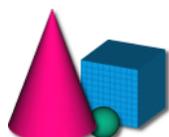


Fig 9 Example of correctly filled window for new material adding.



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It is required to assign the new medium to the box in which the wave propagates. For that purpose pick the *Box* object from the project tree by clicking on it and in the *Data* tab of *Property Editor* (see Fig. 10) click on the *Medium* row. The *Set Medium* option will be available.

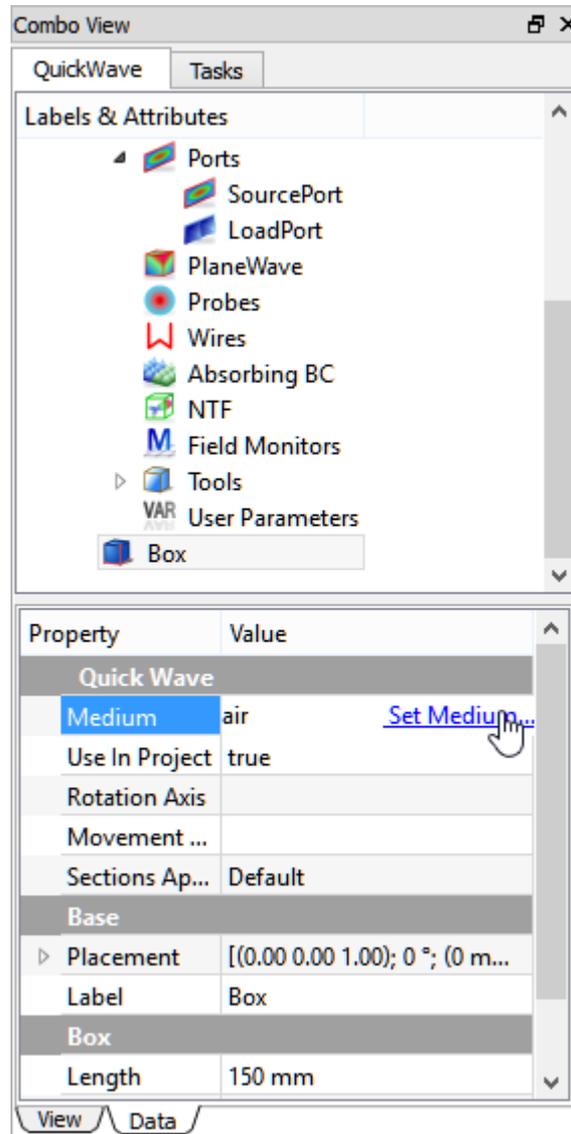
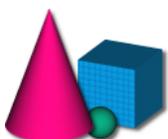


Fig 10 Element properties window of Data tab.

Pressing this option invokes Project Media dialogue (Fig. 11), from where "dielectric" medium should be chosen.



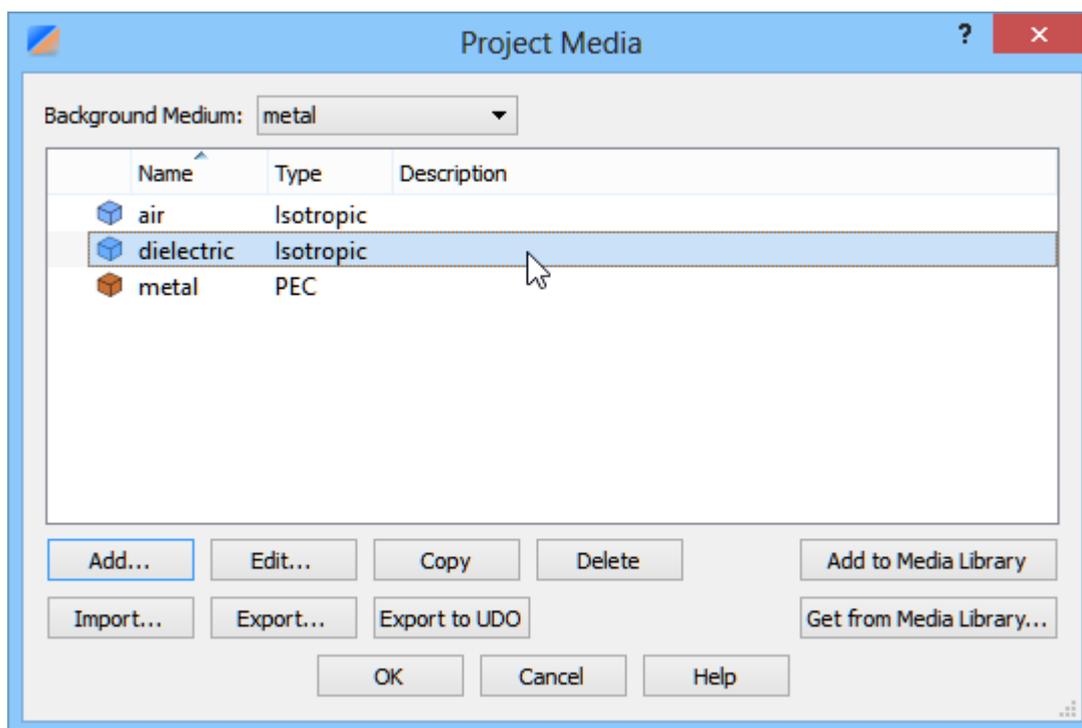


Fig 11 Dialogue window for materials available in the project.



Save the project with a new name and run simulation using **Start** button.

In the *QW-Simulator* measure the wavelength for the considered case. According to the formula given above, the wavelength is equal to $\lambda = \frac{v}{f} = \frac{c}{f\sqrt{\epsilon_r}} = \frac{c}{f\sqrt{4}} = 0.0375m = 37.5mm$. Due to

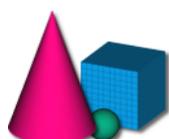
improved space discretisation the precise wavelength value of 37.5 mm is calculated in the simulation.

The display observation allows noticing the change of plane wave speed (*Property 1*). It is advised to check how the speed of a plane wave depends on μ_r change.

Envelope shape in lossy media

In the scenarios considered above the medium in which a plane wave propagated was a lossless medium. The observed fields' envelope was a straight line. In this paragraph a lossy medium will be considered and its influence on envelope shape will be observed.

Open example *ppw4.QWpro* in *QW-Modeller* or modify *ppw1.QWpro* in the following way. Open the *Project Media* dialogue (Fig. 8) as it was done before and add the "dielectric" medium. The window as in Fig 9 will appear. Modify the *Sigma* parameter to be equal to 0.1 (it is expressed in $\frac{S}{m}$). Accept the changes by pressing *OK* button and save the project with a new name.



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The attenuation coefficient α , determining the envelope shape, can be calculated analytically using the following formula for the propagation constant:

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = 18.41 + j85.78$$

$$\text{Real part } \alpha = \text{Re}\{\gamma\} = 18.41$$

The attenuation coefficient may be also calculated from the simulation data in the 1D display window following the below formula:

$$\alpha = \frac{\ln\left(\frac{E(x_1)}{E(x_2)}\right)}{(x_2 - x_1)}$$

where:

- x_1 first marker localisation (in mm) (e.g. at the beginning of guide)
- x_2 second marker localisation (in mm) (e.g. at the end of guide)
- $E(x_1)$ electric field magnitude read for x_1 position
- $E(x_2)$ electric field magnitude read for x_2 position

The display showing exemplary markers' position for attenuation coefficient calculation together with their values is shown in Fig 12.

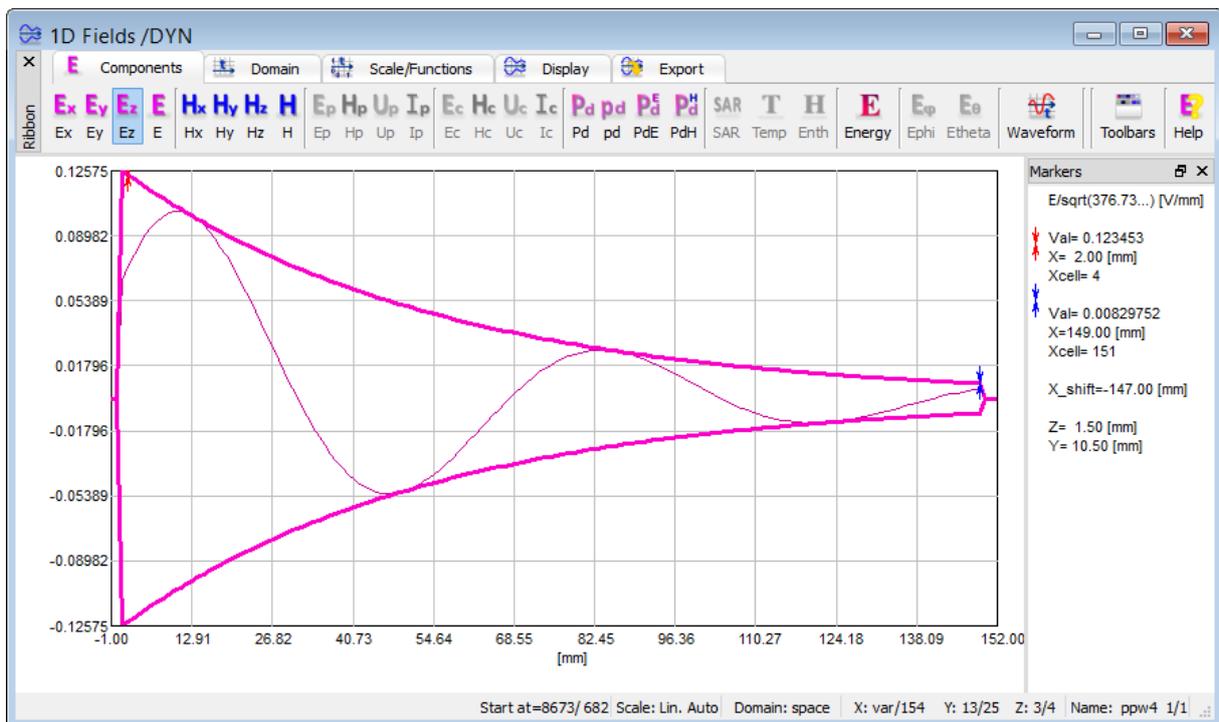
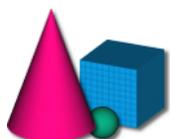


Fig 12 Electric field envelope for lossy medium.

The calculated attenuation coefficient α value is:

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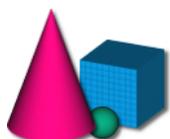
$$\alpha = \frac{\ln\left(\frac{E(x_1)}{E(x_2)}\right)}{(x_2 - x_1)} = 18.37$$

Good agreement between theoretical calculations and simulation data is achieved.

It is recommended to change σ parameter to 1 and observe the envelope shape change.

Summary

In this paragraph it has been shown how to use *QuickWave* software for testing the basic properties of a plane wave.



Plane wave at the media boundary

In previous paragraph, a plane wave propagation in homogeneous, linear and isotropic medium has been discussed. In this paragraph a plane wave incident at the two homogeneous media boundary will be considered. The model used in this paragraph is named *ppw5.QWpro*. It is assumed that the wave incidents normally at the media boundary.

Theory

Fig. 13 shows a situation in which a plane wave incidents at the media boundary. In this case 3 types of waves can be described:

- incident wave, \vec{E}_1^+ , \vec{H}_1^+ ,
- reflected wave, \vec{E}_1^- , \vec{H}_1^- ,
- transmitted wave, \vec{E}_2 , \vec{H}_2

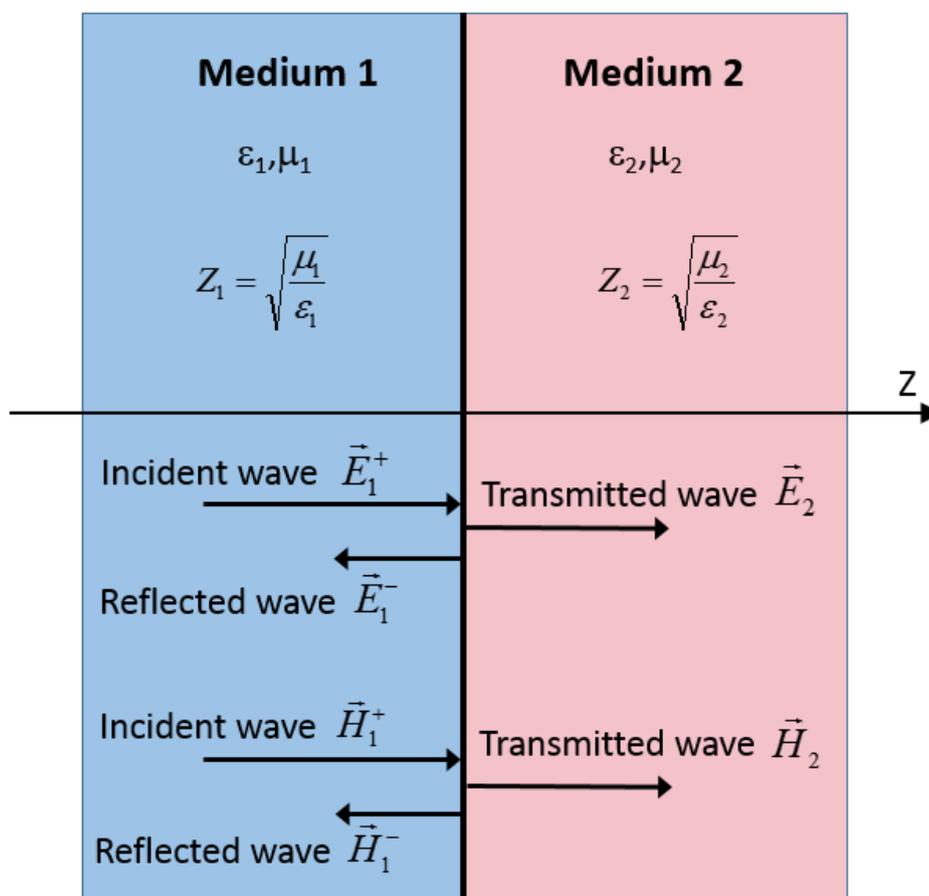
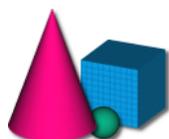


Fig 13 Graphic scheme of normal wave incidence at two mediums border.

In this case a reflection coefficient, describing a reflected to an incident field ratio, can be defined:



$$\Gamma = \frac{\underline{\vec{E}}_1^-(z = z_b)}{\underline{\vec{E}}_1^+(z = z_b)}$$

This allows determining the formulas describing the waves defined above:

Incident wave:

$$\underline{\vec{E}}_1^+ = \vec{i}_x \underline{E}_0 e^{j(\omega t - \beta_1 z)}$$

$$\underline{\vec{H}}_1^+ = \vec{i}_y \frac{E_0}{Z_1} e^{j(\omega t - \beta_1 z)}$$

Reflected wave:

$$\underline{\vec{E}}_1^- = \vec{i}_x \Gamma \underline{E}_0 e^{j(\omega t + \beta_1 z)}$$

$$\underline{\vec{H}}_1^- = \vec{i}_y (-\Gamma) \frac{E_0}{Z_1} e^{j(\omega t + \beta_1 z)}$$

Transmitted wave:

$$\underline{\vec{E}}_2 = \vec{i}_x (1 + \Gamma) \underline{E}_0 e^{j(\omega t - \beta_2 z)}$$

$$\underline{\vec{H}}_2 = \vec{i}_y (1 - \Gamma) \frac{E_0}{Z_1} e^{j(\omega t - \beta_2 z)}$$

The reflection coefficient Γ can be calculated due to the assumption that impedance at the media boundary must be continuous:

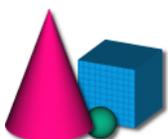
$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

The above wave formulas confirm that in medium 1 incident and reflected waves merge, giving partially standing wave, as constant maxima and minima of fields' envelopes distribution in space exists (a standing wave property) and also the energy is transferred in a direction of an incident wave propagation (typical for traveling wave). In case of medium 2, only traveling wave exists.

The predominant wave type in medium 1 can be determined using the standing wave ratio SWR (or VSWR) defined as the ratio of maximum field envelope (electric or magnetic) to minimum field envelope:

$$SWR = \frac{|E(z)|_{\max}}{|E(z)|_{\min}}$$

SWR is related to a reflection coefficient according to the following formula:



$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Please note that the value of SWR varies in the range (1; ∞). If SWR tends to 1, we deal with a more traveling-like wave.

Paragraph objectives

- 1) Analysis of a reflection coefficient and a standing wave ratio for a plane wave incident at a boundary of two dielectrics
- 2) Analysis of a reflection coefficient and a standing wave ratio for a plane wave incident on a perfect conductor

Tutorials

Two scenarios will be considered: a plane wave incident at a concrete wall and on a flat metal surface.

Normal incidence of a plane wave on the two lossless dielectric media boundary

Model considered herein works at 5GHz, which according to IEEE 802.11 specification, is a wireless communication frequency. The wave propagates in air, incidents on a concrete wall and further propagates in this medium. The reflection coefficient, SWR, reflected and transmitted power will be determined. Theoretical calculations will be compared with the simulation results.

The parameters of a concrete wall are assumed as follows: $\epsilon_{r2}=4.5$, $\mu_{r2}=1$, $\sigma=0$.

Characteristic impedances of the two media are calculated as:

$$Z_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = Z_0 = 120\pi[\Omega]$$

$$Z_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_{r2}\mu_0}{\epsilon_{r2}\epsilon_0}} = \frac{\sqrt{2}}{3} Z_0 = 40\sqrt{2}\pi[\Omega]$$

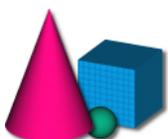
Thus the reflection coefficient equals:

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} \cong -0.36$$

Negative value of the reflection coefficient means that the wave propagates from a medium of higher impedance.

Standing wave ratio in a first medium is calculated as follows:

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} \cong 2.125$$



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The calculated value indicates that the wave in first medium is more like a traveling wave than a standing wave.

The above parameters should be now extracted from the simulation results. Start simulation of the

ppw5.QWpro scenario using  **Start** button in *Simulation* tab. Open 1D visualization window by clicking

 **Fields** button in *1D Fields* tab. Switch to existing field component i.e. *Hy*. The media boundary should be clearly visible ($x = 100 \text{ mm}$). If not, adjust the scale using *Scaling* dialogue available with *Alt + S* keyboard combination. Place markers at the envelope maximum and minimum according to standing wave ratio definition. Standing wave ratio can be now calculated automatically using *Attn* button in *Function* tab of *1D Fields* window. The SWR value is visible in *Markers* tab.

The window display including markers positions is shown in Fig 14.

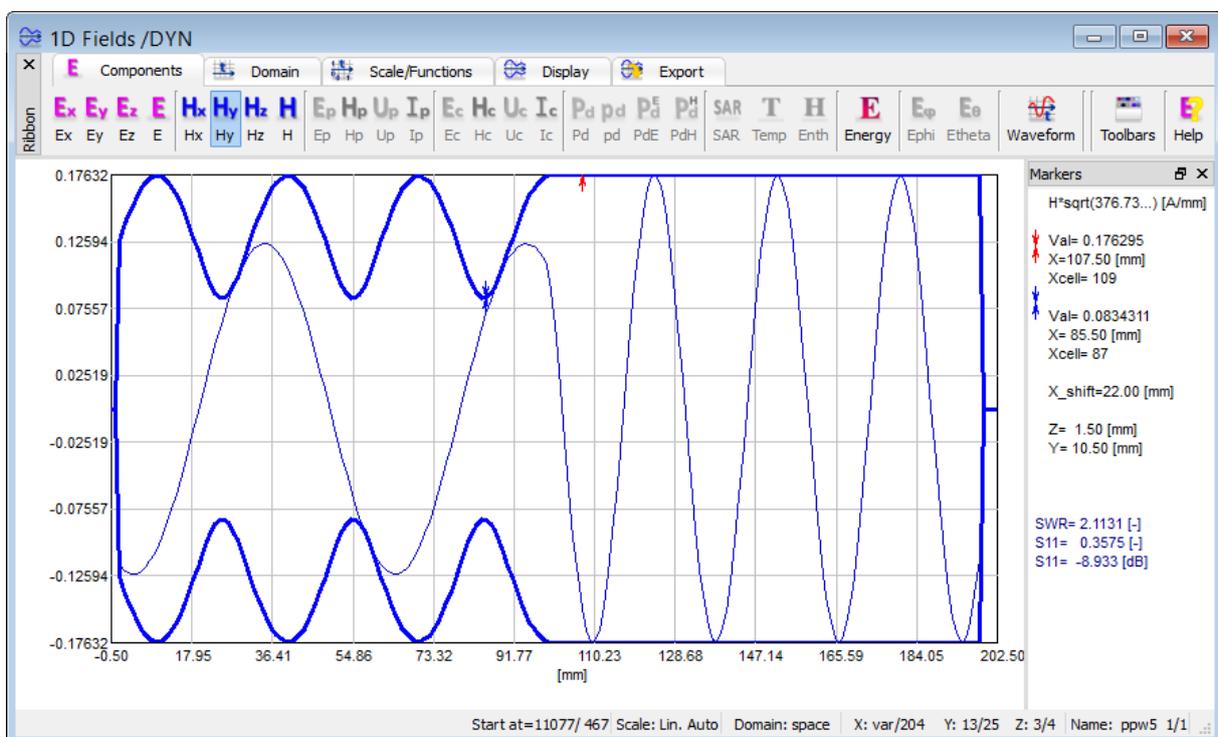
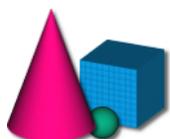


Fig 14 Magnetic field *Hy* element envelope and temporary waveform (example).

The value of SWR calculated from simulation data is very close to the theoretical value. Based on that value, the reflection coefficient can be calculated as follows:

$$|\Gamma| = \frac{SWR - 1}{SWR + 1}$$

The above formula allows calculating an absolute value of the Γ coefficient. To determine the sign of reflection coefficient, the fields' envelope must be observed. If the magnetic field envelope takes maximum value at the media boundary (and respectively, envelope of electric field takes minimum) the wave propagates from a medium of higher impedance (to medium of lower impedance) and the sign of reflection coefficient is minus. If opposite the sign is plus.



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Power reflected from the concrete wall will be now calculated. Assuming that the concrete wall reflects Γ of electric field and Γ of magnetic field, the reflected power is proportional to Γ^2 :

$$\Gamma^2 = 0.36^2 \cong 0.13$$

This means that 13% of a plane wave power is reflected from the concrete wall and 87% is transmitted further through a concrete material.

Normal incidence of a plane wave on a perfect conductor surface

In this case an envelope of a standing wave will be demonstrated. Assuming that a plane wave incidents normally on a perfect metal (*perfect electric conductor – PEC*) with impedance equal to 0, reflection coefficient is equal:

$$\Gamma = \frac{0 - Z_1}{0 + Z_1} = -1$$

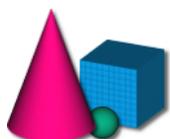
The SWR in this case equals to:

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \rightarrow \infty$$

According to SWR value, the wave in a first medium (air) is a standing wave. No energy is transferred through the media boundary and the fields in envelope nodes have values equal to 0.

The above values and conclusions may be confirmed in electromagnetic simulation. Open the ready to run *ppw6.QWpro* model in *QW-Modeller* or open *ppw5.QWpro* and modify it adequately performing the following:

- Remove the *LoadPort* port since it is not required in this case. Pick the *LoadPort* port on the project tree (see Fig. 15) and remove with *Delete* option available under right mouse button.
- Change the medium of section 2 to metal by editing *Medium* parameter of the *section2* object (see Fig. 16).
- Save the project with a new name.



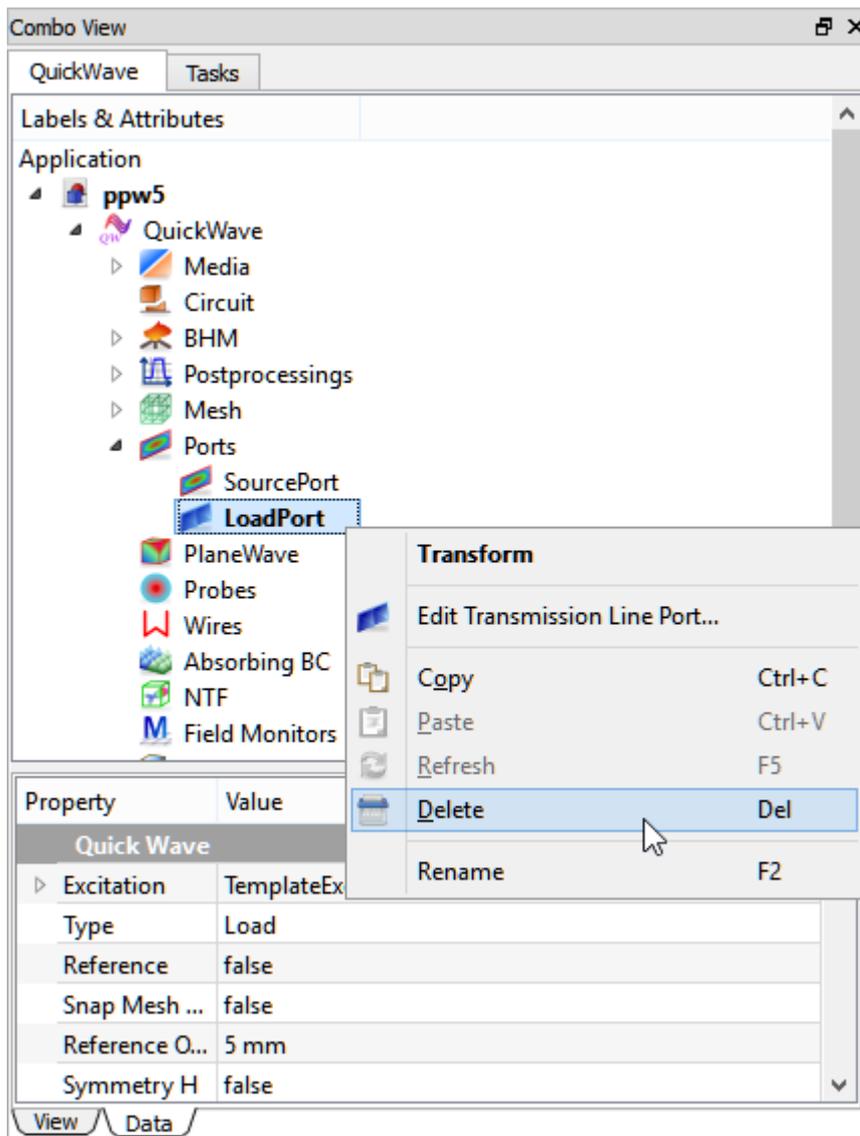
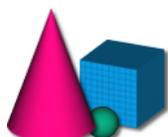


Fig 15 How to delete port (element) from the project.



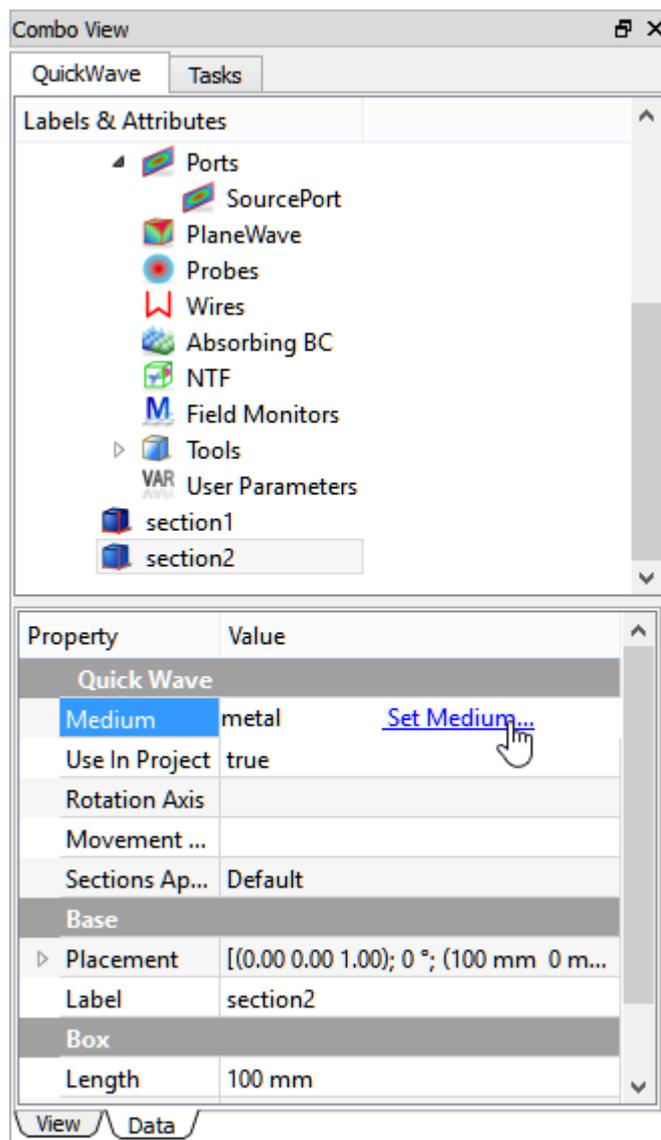
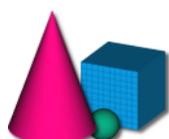


Fig 16 Element properties edition.

Run the simulation by pressing  **Start** button in *Simulation* tab. Open 1D display window using  **Fields** button in *1D Fields* tab. Switch to existing field component i.g. *Hy*. Fig. 17 shows the display that should be seen.



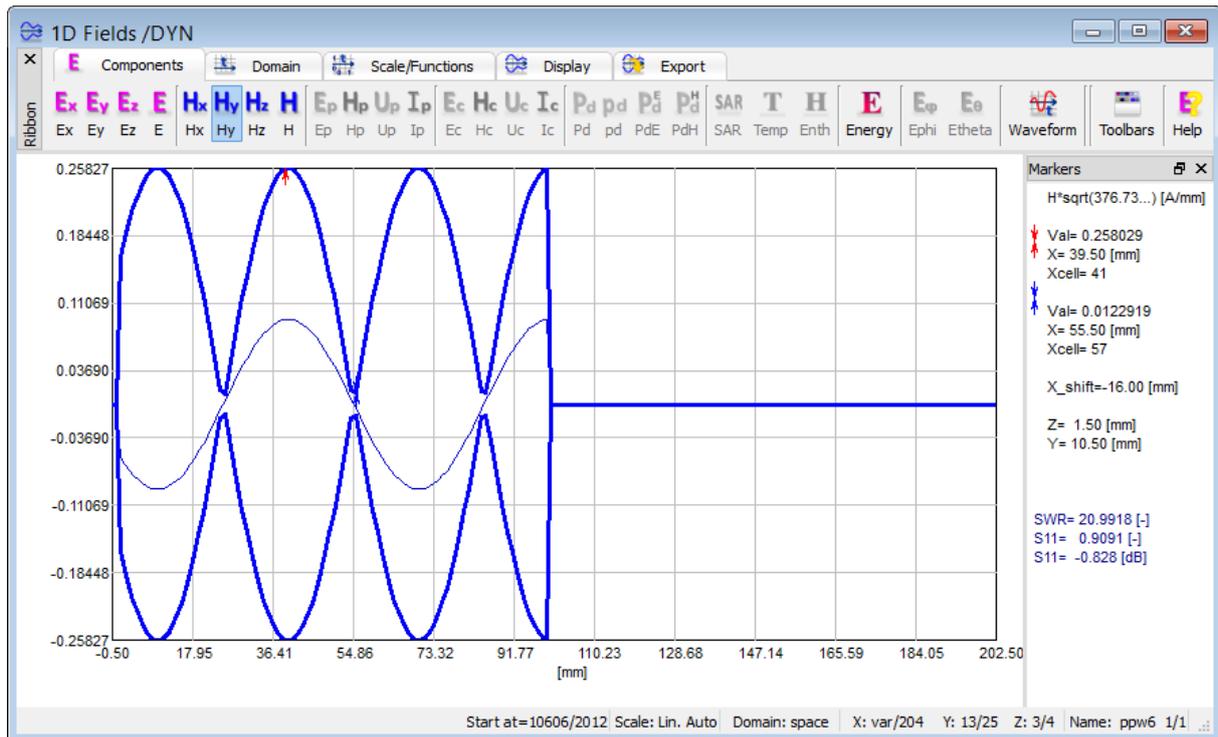
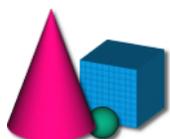


Fig 17 Magnetic field envelope of a wave normal to PEC surface.

It can be seen that there is no propagation in perfect metal. The values of magnetic field in the envelope nodes are close to 0. The values are not 0 due to the numeric accuracy limit – the smaller the mesh cell dimension will be, the closer field value to 0 would be.

Summary

In his paragraph the fields' envelopes of a plane wave, incident on different media boundary was demonstrated. It is recommended to investigate the EM fields' envelopes in case when a plane wave propagates from a medium of lower impedance and a plane wave incidents perpendicularly on a lossy dielectric.



Plane wave at the three media layered structure

Plane wave incident on three media boundary is the most interesting and complex case. The definitions of matching, quarter wavelength transformer and half wavelength transformer will be introduced and explained.

Theory

The media structure discussed in his paragraph is shown in Fig 18.

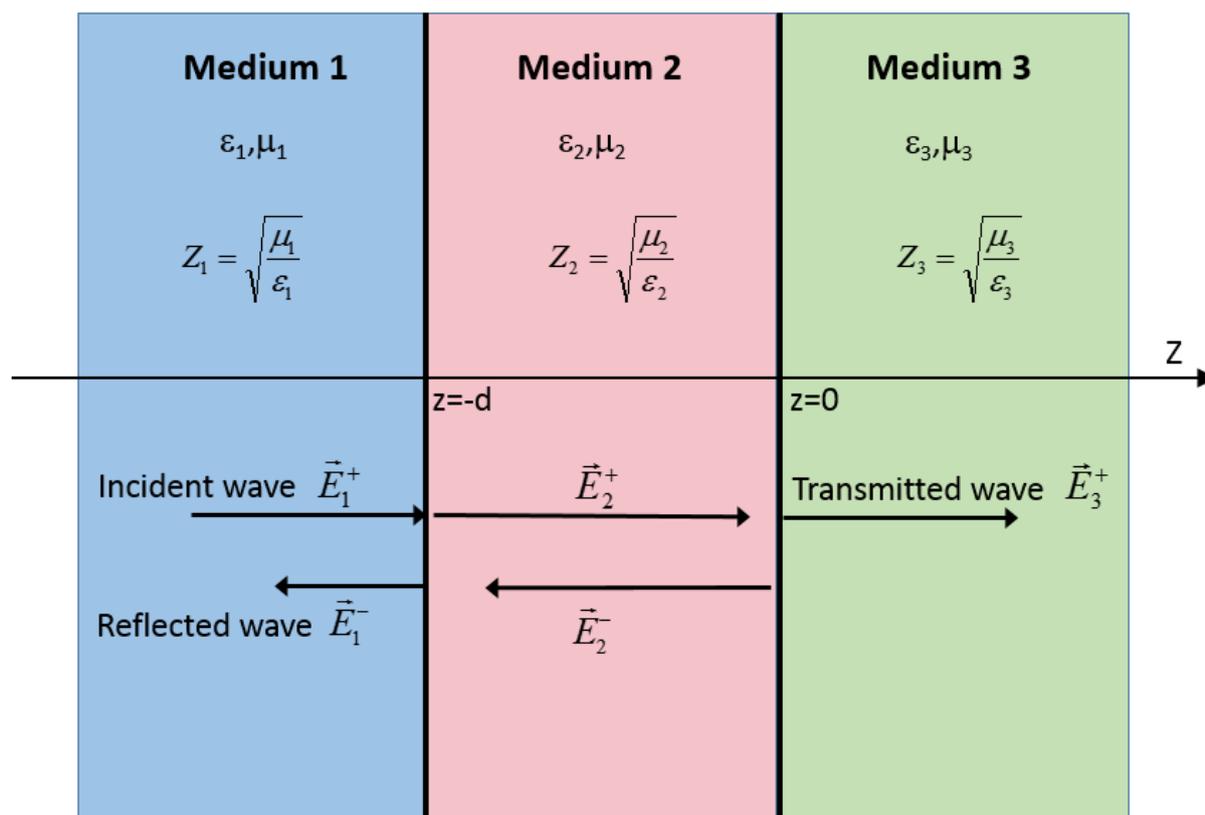


Fig 18 3-medium structure and normal plane wave incident.

In the considered case a plane wave propagating in medium 1 incidents at the boundary between medium 1 and 2, it is partially reflected (according to the rules described before) and then propagates further in medium 2 until it reaches the boundary of medium 3. At the medium 3 boundary a plane wave partially reflects and the remaining wave further propagates in medium 3. The wave reflected from the boundary of medium 2 and medium 3, propagating in medium 2, is partially transmitted to medium 1, thus it influences the amount of total reflection seen in medium 1.

The above reflections scenario can be simplified to a scenario of a plane wave incident at the boundary of two different media. It is done by introducing a variable describing the impedance of medium 3 transformed by a slab of medium 2 (it is an impedance “seen” by a wave incident at medium 2 boundary). This wave impedance is defined with the following formula:



$$Z_{w2}(z) = Z_2 \frac{Z_3 - jZ_2 \operatorname{tg}(\beta_2 z)}{Z_2 - jZ_3 \operatorname{tg}(\beta_2 z)}$$

giving:

$$Z_{w2}(-d) = Z_2 \frac{Z_3 - jZ_2 \operatorname{tg}[\beta_2 \cdot (-d)]}{Z_2 - jZ_3 \operatorname{tg}[\beta_2 \cdot (-d)]} = Z_2 \frac{Z_3 + jZ_2 \operatorname{tg}(\beta_2 d)}{Z_2 + jZ_3 \operatorname{tg}(\beta_2 d)}$$

The reflection coefficients are calculated for both boundaries as follows:

- Reflection coefficient $\Gamma_{2,3}$ (between medium 2 and 3)

$$\Gamma_{2,3} = \frac{Z_3 - Z_2}{Z_3 + Z_2}$$

- Reflection coefficient $\Gamma_{1,2}$ (between medium 1 and 2)

$$\Gamma_{1,2} = \frac{Z_{w2}(-d) - Z_1}{Z_{w2}(-d) + Z_1}$$

The corresponding standing wave ratios are calculated as:

$$SWR_1 = \frac{1 + |\Gamma_{1,2}|}{1 - |\Gamma_{1,2}|}, \quad SWR_2 = \frac{1 + |\Gamma_{2,3}|}{1 - |\Gamma_{2,3}|}$$

It is worth noting that in specific cases, $\operatorname{tg}(\beta_2 d)$ component in the formula for transformed impedance equals 0 or ∞ . Those situations will be considered below.

Quarter wavelength transformer

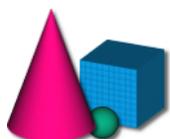
When the thickness (d) of medium 2 is equal to $\frac{1}{4}$ of a wavelength in this medium ($d = \frac{\lambda_2}{4}$), $\operatorname{tg}(\beta_2 d)$ component equals to:

$$\operatorname{tg}(\beta_2 d) = \operatorname{tg}\left(\frac{2\pi \lambda_2}{\lambda_2} \frac{\lambda_2}{4}\right) = \operatorname{tg}\left(\frac{\pi}{2}\right) = \infty$$

The formula for transformed impedance simplifies to:

$$Z_{w2}(-d) = \frac{(Z_2)^2}{Z_3}$$

Please note that the *tangent* function is periodic with period equal to π , so the above formula is correct for medium slab thicknesses of $\frac{\lambda_2}{4} + n \cdot \frac{\lambda_2}{2}$, where n is an absolute positive number.



Half wavelength transformer

When the thickness (d) of medium 2 is equal to half of a wavelength in this medium ($d = \frac{\lambda_2}{2}$), $tg(\beta_2 d)$ component equals to:

$$tg(\beta_2 d) = tg\left(\frac{2\pi}{\lambda_2} \frac{\lambda_2}{2}\right) = tg(\pi) = 0$$

The formula for transformed impedance simplifies to:

$$Z_{w2}(-d) = Z_3$$

In this case the above formula is correct for slab thicknesses of $n \cdot \frac{\lambda_2}{2}$, where n is an absolute positive number.

Impedance matching

The wave transformers introduced above are commonly used for impedance matching of different media. This means that the parameters of medium 2 are chosen to assure no reflections in medium 1.

Quarter wavelength transformer allows matching media of different impedances providing that the impedance of the transformer material equals:

$$Z_2 = \sqrt{Z_1 \cdot Z_3}$$

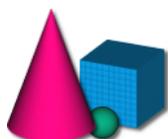
Half wavelength transformer is transparent for the wave as long as its thickness equals to $n \cdot \frac{\lambda_2}{2}$ at a chosen frequency.

Paragraph objectives

- 1) Analysis of the fields' envelopes for three media layered structure.
- 2) The design of impedance matching in three media layered structure.

Tutorials

The scenario with a concrete wall considered before is revert and it will be demonstrated how to accomplish an impedance matching for air-concrete-air structure using a quarter wavelength transformer.



Plane wave incident normally at the layered structure of three lossless media: air-concrete-air
 In the previous paragraph the reflection of a plane wave incident at the air-concrete boundary was considered. Assume more practical situation when a plane wave incidents at the layered structure of air-concrete-air, which corresponds to a scenario where WiFi signal is crossing room's wall.

For calculation simplicity, it is assumed that the relative permittivity of concrete equals to 4. The thickness of a room's wall is 7.5 mm. A plane wave incidents at the dielectric layered structure under the following conditions:

$$Z_{w2}(-d) = Z_2 \frac{Z_3 + jZ_2 \operatorname{tg}(\beta_2 d)}{Z_2 + jZ_3 \operatorname{tg}(\beta_2 d)}$$

$$Z_{w2}(-d) = Z_2 \frac{Z_3 + jZ_2 \operatorname{tg}\left(\frac{\pi}{2}\right)}{Z_2 + jZ_3 \operatorname{tg}\left(\frac{\pi}{2}\right)} = \frac{(Z_2)^2}{Z_3} = \frac{Z_0}{4}$$

Thus:

$$\Gamma_{1,2} = \frac{Z_{w2}(-d) - Z_1}{Z_{w2}(-d) + Z_1} = -\frac{3}{5}$$

$$\Gamma_{2,3} = \frac{Z_3 - Z_2}{Z_3 + Z_2} = \frac{1}{3}$$

And:

$$SWR_1 = \frac{1 + |\Gamma_{1,2}|}{1 - |\Gamma_{1,2}|} = 4, \quad SWR_2 = \frac{1 + |\Gamma_{2,3}|}{1 - |\Gamma_{2,3}|} = 2$$

The wall thickness was chosen so that it equals to a quarter wavelength in a concrete material for excitation frequency of 5GHz. This does not assure the impedance matching because the $Z_2 = \sqrt{Z_1 \cdot Z_3}$ condition is not obeyed. In this case a partially standing wave in medium 1 is observed and reflections on the concrete wall boundaries are observed. Signal propagation through several walls can significantly decrease signal power and the reception will be not possible.

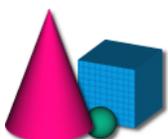
To investigate the field's envelope for the considered layered structure open *ppw7.QWpro* model in



QW-Modeller and start simulation using *Start* button in the *Simulation* tab. Open 1D display window



using *Fields* button in *1D Fields* tab of *QW-Simulator*. Switch to existing electric field component i.e. *Ez*. Fig. 19 shows the display that should be seen.



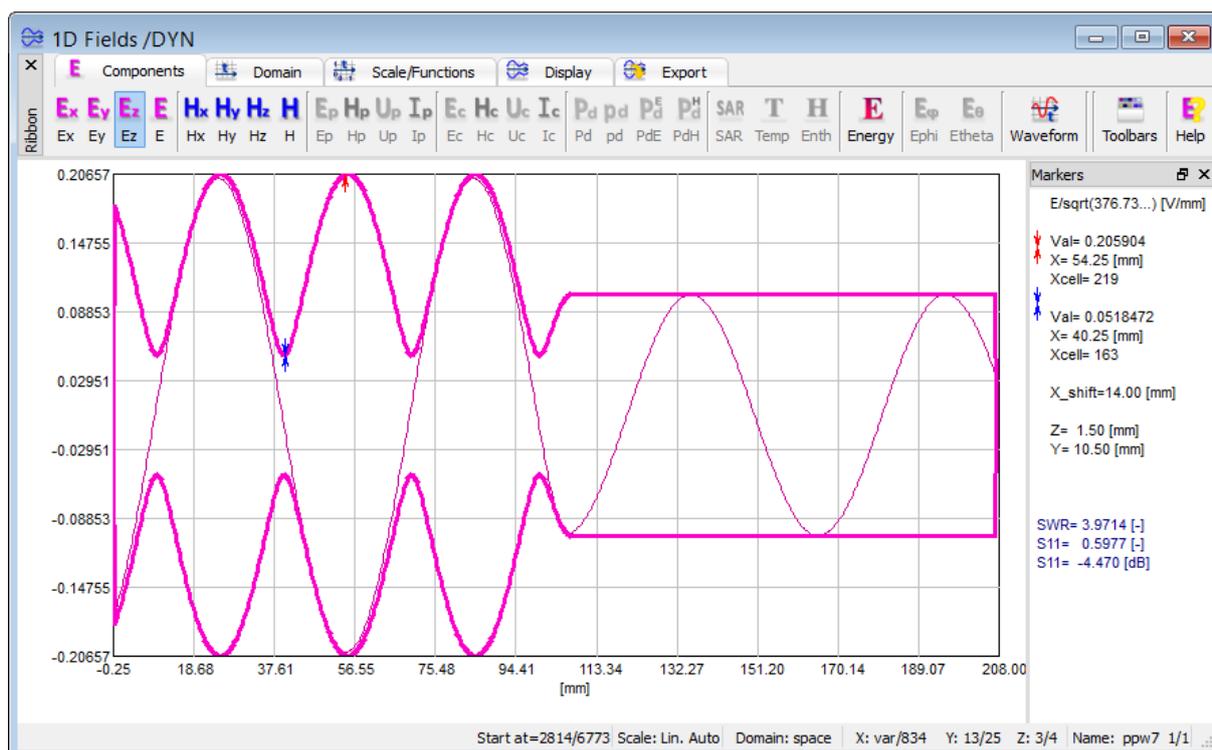


Fig 19 Electric field envelope and temporary waveform of 3- media example.

The SWR value in medium 1 calculated from simulation data confirms the theoretical value. Similar calculations should be performed for medium 2.

Plane wave incident normally at the layered structure of three lossless media: air-concrete-air – impedance matching

The reflections in medium 1 can be eliminated when the impedance matching is assured. Since the impedances of medium 1 and 3 are equal, the half wavelength transformer (transparent for the wave) is considered. The thickness of a concrete wall needs to be changed to $\frac{\lambda_2}{2}$ and the following value of transformed impedance is obtained:

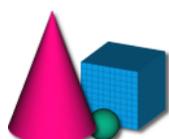
$$Z_{w2}(-d) = Z_2 \frac{Z_3 + jZ_2 \operatorname{tg}(\beta_2 d)}{Z_2 + jZ_3 \operatorname{tg}(\beta_2 d)}$$

$$Z_{w2}(-d) = Z_2 \frac{Z_3 + jZ_2 \operatorname{tg}(\pi)}{Z_2 + jZ_3 \operatorname{tg}(\pi)} = Z_3 = Z_0$$

Thus:

$$\Gamma_{1,2} = \frac{Z_{w2}(-d) - Z_1}{Z_{w2}(-d) + Z_1} = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$$

$$\Gamma_{2,3} = \frac{Z_3 - Z_2}{Z_3 + Z_2} = \frac{1}{3}$$



And:

$$SWR_1 = \frac{1+|\Gamma_{1,2}|}{1-|\Gamma_{1,2}|} = 1, \quad SWR_2 = \frac{1+|\Gamma_{2,3}|}{1-|\Gamma_{2,3}|} = 2$$

According to the above calculations, using 15 mm thick wall should eliminate reflections. To confirm this with electromagnetic simulation the *ppw8.QWpro* should be opened or *ppw7.QWpro* must be modified as follows: It is required to increase the wall thickness. On the project tree pick *section2* object and change its length in the *Property Editor* (see Fig. 20) to 15 mm. At the same time the length of *section3* object needs to be decreased by 7.5 mm and its position in X direction needs to be shifted by the same value. The correct settings are shown in Fig. 21.

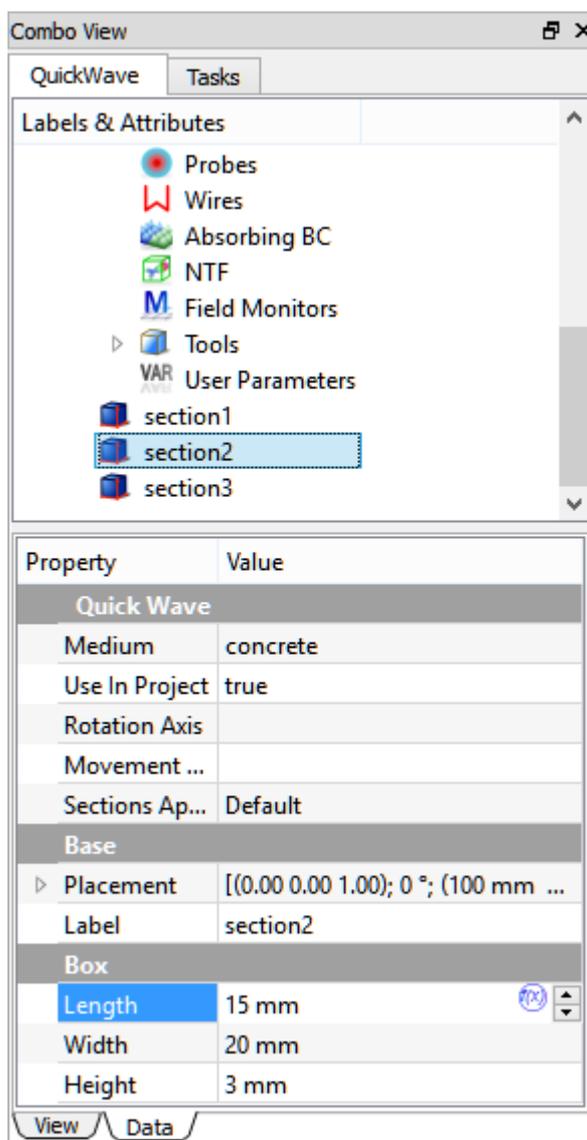
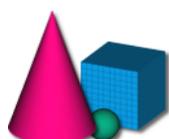


Fig 20 Properties of section2 representing concrete wall.



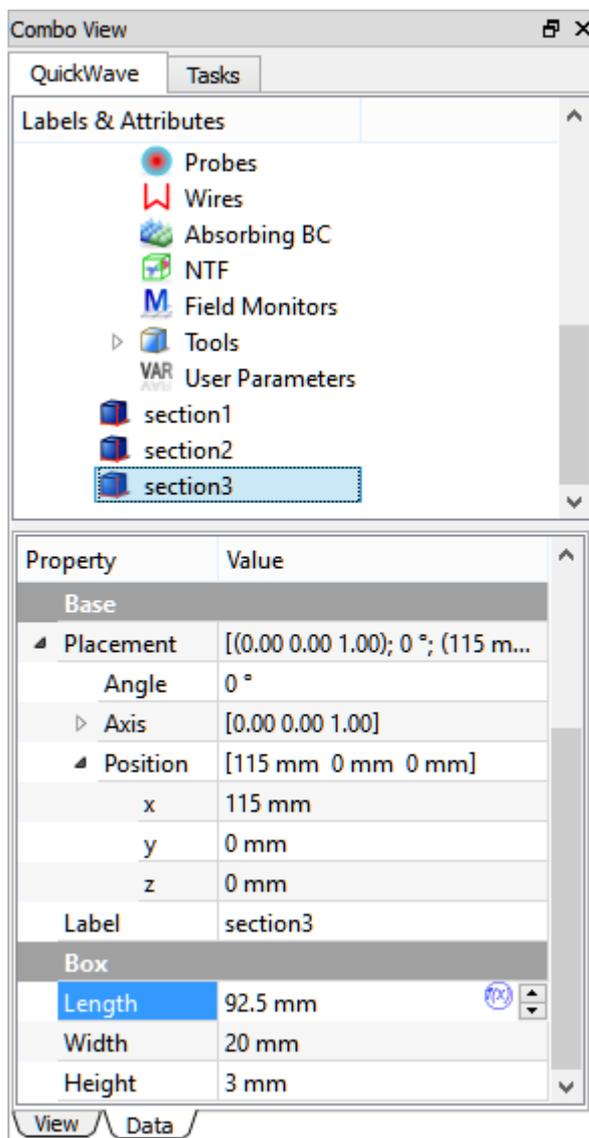
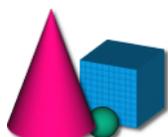


Fig 21 Correct parameters after section3 modification.

Save the project with a new name and run simulation by pressing  **Start** button in *Simulation* tab.

Open 1D display window by pressing  **Fields** button. Switch the view to existing electric field component i.e. E_z . Fig. 22 shows the display that should be seen.



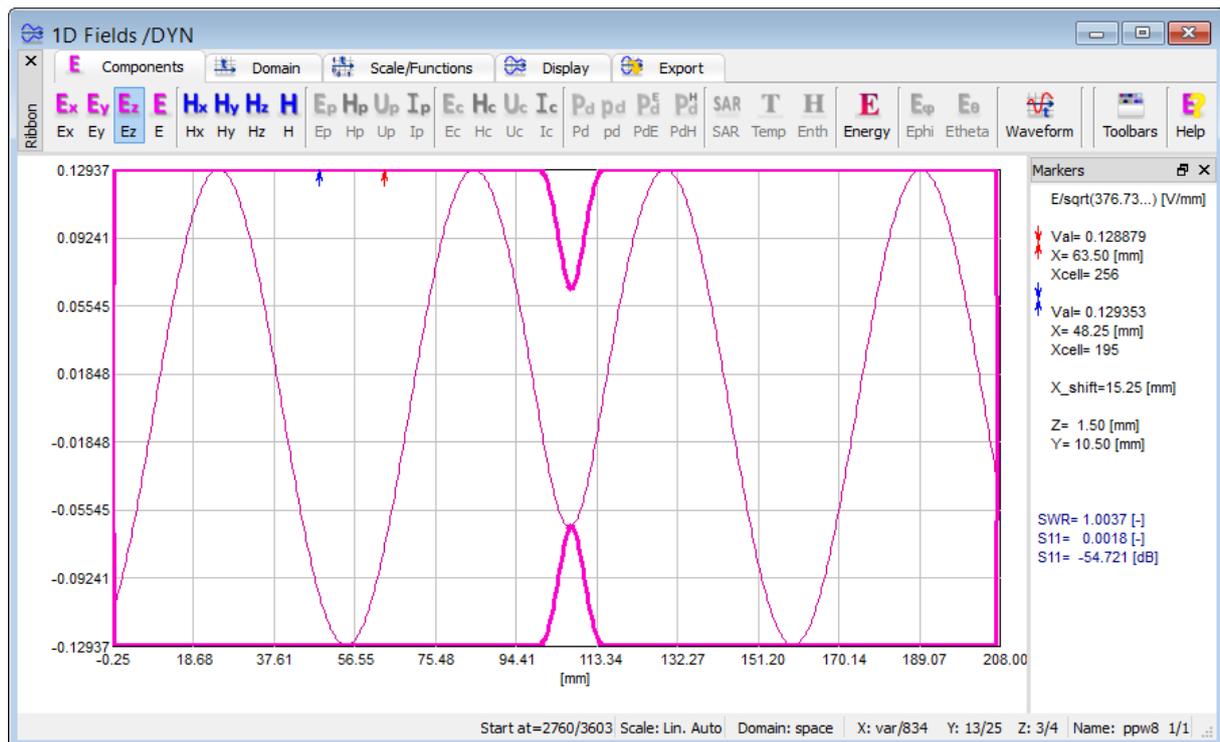


Fig 22 Matching realized with half wave transformer.

Simulation results have confirmed the theoretical calculations that using a half wavelength transformer assures the impedance matching in the considered scenario.

Summary

In this paragraph the possibilities of matching a dielectric layered structure has been demonstrated. It is recommended to design an impedance matching for 5GHz between air (medium 1) and a dielectric with relative permittivity $\epsilon_r=9$ (medium 3) using quarter wave transformer. All three mediums have permeability $\mu_r=1$. The medium parameters of medium 2 should be chosen appropriately.

